A RIMS - Kyoto University &

"Arithmetic and Homotopic Galois Theory" lecture

BERKOVICH METHODS FOR ANABELIAN RECONSTRUCTIONS AND THE RESOLUTION OF NONSINGULARITIES

E. LEPAGE - April. 08, 10, & 12, 2024

Berkovich spaces are an approach to analytic geometry over non-archimedean fields. Compared to other approaches, such as rigid spaces or Huber's adic spaces, they tend to have nicer topological properties. The topology of Berkovich spaces is strongly related to the combinatorics of models over the ring of integers. The recovery of such combinatorics from fundamental groups often plays an important role in anabelian geometry. In these lectures, I will try to explain how Berkovich spaces and analytic methods can be helpful in anabelian geometry.
Keywords: p-adic analytic geometry, Berkovich spaces, anabelian geometry

REFERENCES

[Ber90] V. G. Berkovich. *Spectral Theory and Analytic Geometry over Non-Archimedean Fields*. Vol. 33. Mathematical Surveys and Monographs. Providence: American Mathematical Society, 1990.

[MT23] S. Mochizuki and S. Tsujimura. *Resolution of Nonsingularities, Point-theoreticity, and Metric-admissibility for p-adic Hyperbolic Curves*. RIMS preprint 1974. 2023. URL: `https://www.kurims.kyoto-u.ac.jp/preprint/file/RIMS1974.pdf`.

GEOMETRY OF BERKOVICH SPACES — TALK 1

In this talk, I will explain the definition of Berkovich spaces and explain its basic properties. I will state some general applications of the theory. I will also discuss their relation to other approaches of analytic geometry over non-archimedean fields and give a more precise description of curves.

Keywords: Berkovich spaces, p-adic analytic geometry, geometry of curves.

REFERENCES

[Ber90] V. G. Berkovich. *Spectral Theory and Analytic Geometry over Non-Archimedean Fields*. Vol. 33. Mathematical Surveys and Monographs. Providence: American Mathematical Society, 1990.

[Ber93] V. G. Berkovich. "Étale cohomology for non-Archimedean analytic spaces". In: *Publication mathématiques de l'Institut des hautes études scientifiques* 78 (1993), pp. 5–161.

[Duc] A. Ducros. *La structure des courbes analytique*. URL: `https://webusers.imj-prg.fr/~antoine.ducros/trirss.pdf`.

[Duc07] A. Ducros. "Espaces analytiques p-adiques au sens de Berkovich". fr. In: *Séminaire Bourbaki : volume 2005/2006, exposés 952-966*. Astérisque 311. talk:958. Société mathématique de France, 2007, pp. 137–176. URL: `http://www.numdam.org/item/SB_2005-2006__48__137_0/`.

RESOLUTION OF NON-SINGULARITIES AND LOG-DIFFERENTIALS — TALK 2

This talk will focus on Mochizuki and Tsujimura's proof of the absolute anabelian conjecture: every isomorphism between the étale fundamental groups of hyperbolic curves over finite extensions of $\mathbb{Q}_p$ is geometric. The new input of their work is the proof of resolution of non-singularities: given a hyperbolic curve X over a finite extensions of $\mathbb{Q}_p$ is geometric, every divisorial valuations on $K(X)$ comes from some irreducible component of the special fiber of the stable model after replacing X by some finite étale cover. If Mochizuki and Tsujimura's proof is written in a purely scheme-theoretic framework, some of its intuition comes from previous work using analytic methods: resolution of non-singularities can be reduced to the study of the vanishing of differentials appearing in the image of the Hodge-Tate map $H^1(X_{\mathbb{C}_p}, \mathbb{Z}_p(1)) \to H^0(X_{\mathbb{C}_p}, \Omega^1)$. I will reformulate their proof using analytic geometry.

Keywords: p-adic anabelian geometry, absolute anabelian conjecture, resolution of non-singularities, vanishing of log-differentials.

References

[Lep23] E. Lepage. *Resolution of non-singularities and the absolute anabelian conjecture*. 2023. URL: https://arxiv.org/abs/2306.07058.

[Lep13] E. Lepage. "Resolution of nonsingularities for Mumford curves". In: *Publ. Res. Inst. Math. Sci.* 49.4 (2013), pp. 861–891.

[MT23] S. Mochizuki and S. Tsujimura. *Resolution of Nonsingularities, Point-theoreticity, and Metric-admissibility for p-adic Hyperbolic Curves*. RIMS preprint 1974. 2023. URL: https://www.kurims.kyoto-u.ac.jp/preprint/file/RIMS1974.pdf.

Anabelian reconstruction of metric on Berkovich curves — Talk 3

In this talk, I will keep on studying the differentials in the image of the Hodge-Tate map introduced in the previous talk. I will explain how the tropicalization of these differentials and a natural metric on the Berkovic space can be recovered from the geometric tempered fundamental group of a hyperbolic curve.

Keywords: Tempered fundamental group, log-differentials, metric on Berkovich curves.

References

[Lep] E. Lepage. *Anabelian tempered metric-admissibility*. in preparation.

Schedule Location: RIMS Kyoto University, room 110.

- Talk 1: April 8th (Mon.) 13:00-15:00
- Talk 2: April 10th (Wed.) 10:00-12:00
- Talk 3: April 12th (Fri.) 13:00-15:00

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